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Partial inelasticities from the dual parton model

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Abstract. The question of the energy behavior of the partial inelasticities is studied in the context of the dual parton model. A simple analytical expression is derived which describes the behavior of the partial inelasticities at high energies. A comparison with the results of some other models is also given. The question of the violation of Feynman scaling is considered with reference to the inelasticity problem.

1 Introduction

For many practical calculations of propagation and interactions of hadrons the inelasticity parameter is very important. Its importance is also related with the results of collider and cosmic ray experiments at high energies. The inelasticity gives us information about energy losses of projectile during the collision. Since a full knowledge of all inclusive distributions and their energy dependence is unavailable, a concept of the inelasticity coefficient is of particular value. In large measure this is appropriate for the cosmic ray physics as the inelasticity coefficient shows how the energy is shared between the leading particle and the secondary particles and therefore determines the scales of cosmic ray attenuation and longitudinal development of extensive air showers in the atmosphere. Nevertheless, the energy dependence of the inelasticity is still in question.

The inelasticity K_{tot} is usually defined as the fraction of energy carried away by all new produced secondaries and it can be evaluated from the energy of leading particles. However, in accelerator experiments the inelasticity has not been determined at energies higher then *ISR* [1] because of the difficulties at high energies of measurements of leading particles emitted at very small angles. The point is that collider experiments study mostly the central region of the hadronic interactions: owing to the special technical features of colliders, there are restrictions on minimal angle at which particles can be registered and so at high energies, when many particles (including leading particles) emerge in the very forward cone, the fragmentation region can not be studied properly in collider experiments. Nonetheless, the *SppS*-experiments still gave some information about the fragmentation region: the *UA5* Collaboration [2–4] and the *UA7* Collaboration [5] confirmed a validity of an approximate Feynman scaling in this region.

In principle, the behavior of the inelasticity parameter at high energies can be extracted from several types of cosmic ray data [6–8].

i) γ-families in emulsion chambers at mountain altitudes. Here one can study the attenuation length of the family events and the energy spectrum of hadrons in the families [9]. The conclusions about the energy behavior of the inelasticity, however, are rather sensitive to the assumptions of the chemical composition of the primaries [6].
ii) Extensive air shower data of Akeno and Fly's Eye arrays. Coupled analysis of the shower maximum measurements from *Fly's Eye* and zenith-angle distributions of showers at fixed altitude, from *Akeno,* gives a possibility to determine $K_{tot}(E)$ and $\sigma_{in}^{(p-air)}(E)$. This analysis seems to favor the conclusions about the inelasticity decreasing as the energy increases [7,8]. But on the base of the data in the work [10] the opposite deductions were made as well.
iii) Muon bundles in underground detectors [6]. The underground multi-muon data are sensitive to the chemical composition and the multiplicity in the hadronic interactions of the primary cosmic rays in the atmosphere, as well as to the inelasticity. To disentangle the effects a very careful analysis of these data should be performed.
iv) Study of the hadronic and electromagnetic components of cosmic rays. Here one can study the energy spectrum of hadrons at different depths in the atmosphere and the altitude dependence of the inelasticity. According to the works [11–15], the data on the altitude dependence of the inelasticity of the electromagnetic component and on the energy spectrum of the nucleonic component suggest (at least, at high energies) the inelasticity not decreasing with the rise of energy.

As one can see from this short review, by now there is no definite conclusion regarding the energy dependence of the inelasticity coefficient, in spite of the numerous data from collider and cosmic ray experiments. One could expect that the employment of reliable theoretical models will help to get out of this difficulty. At the same time, the *QCD*, the best candidate for the theory of strong interactions, cannot be of help here because of the dominance of soft processes.

At different times many various theoretical models were developed to provide descriptions of soft hadronic interactions. But the situation here is just the reflection of the experimental state. There are three large groups of the models which differ by the inelasticity behavior predictions [16,6].

a) Models predicting a steep decrease of the inelasticity. Those are such models as, for example, statistical model [17], geometric model [18], Wdowczyk-Wolfendale model [19–22], hydrodynamic model [23,24], modified dual parton model [25,26], valon-gluon model [27,8] and fire-tube model [28,29].

b) Models with a moderate rise of the inelasticity. Here let us cite as examples mini-jet model [30,31] (see, however, the work [29] where a decreasing inelasticity was also obtained in the context of this model), interacting gluon model (after including a semihard component to the original version) [32,33] and quark-gluon string model [34,35].

c) Models giving rise to a rapid increase of the inelasticity. A glowing example of this class of models is the QCD Pomeron model [36]. A glowing example of this class of models is the QCD Pomeron model [36].

In the present paper we study the energy dependence of the partial inelasticities in the framework of the dual parton model (DPM). This model (see, for instance, [37–43]) was developed and extended over the past fifteen years and now it is successfully used for describing the inclusive hadronic spectra in a wide range of energies. The model proved to be able not only to suggest some theoretical basis for understanding a great deal of soft hadronic physics phenomena, but also to give certain predictions for high and very high energies. The model provides a fairly good description of experiment both in the central region and fragmentation regions. It reproduces the violation of Feynman scaling, observed in experiment.

In this work we focus our attention not only on calculations of the inelasticity coefficient in the given model, but also upon the question of the mechanisms which lead to the inelasticity with an energy behavior peculiar to this model. Simple analytical expressions are derived which describe the behavior of the partial inelasticities at high energies.

2 Partial inelasticities

Along with the total inelasticity, the partial inelasticities are often used in practice, which are, by definition, the fractions of energy carried over by the secondaries of a given sort i

$$K_i = \int_0^1 \bar{x}\, \frac{dN_i\left(x;\ \sqrt{s}\right)}{dx}\, dx = \frac{2}{\sqrt{s}} \int_0^{y_{\max}} E^*\left(y;\ \sqrt{s}\right) \frac{dN_i\left(y;\ \sqrt{s}\right)}{dy}\, dy. \quad (2.1)$$

Here $x = P^*/P_0^*$ is Feynman's variable,

$$\bar{x} = \sqrt{x^2 + \frac{4\left\langle m_\perp \right\rangle^2}{s}}, \quad (2.2)$$

(we keep in mind that at high energies $P_0^* \approx \sqrt{s}/2$), $m_\perp$ is the transverse mass of a produced hadron, $y = arsinh\left(xP_0^*/\left\langle m_\perp\right\rangle\right)$ is the hadron rapidity and $\frac{dN_i}{dy}$ is the inclusive spectrum of particles i. It is seen from (2.1) that in order to calculate the partial inelasticities over a wide region of energies, one has to know the inclusive spectrum both for the corresponding broad range of primary energies and for the full rapidity range. From collider data it is evident that a strong violation of Feynman scaling is observed in the central region over a full energy range of modern accelerators and so the energy dependence of the inclusive spectra has to be taken into account at any calculations of $K_i\left(\sqrt{s}\right)$. It is clear that in the case of the rigorous scaling the inelasticity parameter has to be a constant. In contrast, a strong violation of Feynman scaling could lead to an appreciable energy dependence of the inelasticity.

Some questions concerning the violation of the Feynman scaling law were studied in the context of the DPM in our previous works [44,45]. It was found (see [44]) that at high energies the rise of the so-called plateau height $\frac{dN}{dy}\mid_{y=0}$ can be described by a simple relation

$$\frac{dN}{dy}\mid_{y=0} = \left\langle n\left(s\right)\right\rangle \cdot a \cdot \left(1 - \frac{b}{\ln\left(s/s_0\right)}\right), \quad (2.3)$$

where a, b, s_0 are some constants. The interaction of hadrons is considered in the DPM as a production of colored chains stretched between the colliding particles, with the subsequent fragmentation of the chains into the secondary hadrons. The term $\left\langle n\left(s\right)\right\rangle$ in (2.3) denotes the average number of the produced chains, and its energy dependence is as follows:

$$\left\langle n\left(s\right)\right\rangle \simeq \left(\frac{s}{s_0}\right)^{\alpha'}, \quad (2.4)$$

where $\alpha' = \alpha_P - 1 \approx 0.11$ (α_P is the Pomeron intercept) [41]. The expression $\left(1 - \frac{b}{\ln(s/s_0)}\right)$ in the right-hand side of (2.3) describes the violation of scaling for an individual chain. This effect is due to the internal motion of quarks in a hadron [45].

It was also found out [44,45] that for x satisfying the condition (we will call it "the fragmentation region condition")

$$x > \sqrt{\frac{\left\langle m_\perp\right\rangle}{2\sqrt{s}}}, \quad (2.5)$$

one comes to the fragmentation region behavior with relatively small violation of scaling.

Now we can apply these results to the problem of the energy dependence of the partial inelasticities. It is easy to see that the partial inelasticity can also be written in the following form:

$$K_i = \int_0^1 \frac{dN_i\left(y\left(x\right);\ \sqrt{s}\right)}{dy}\ dx. \tag{2.6}$$

Having in mind expression (2.5), let us divide the interval of integration into the central and fragmentation subintervals

$$K_i = \int_0^{x_0} \frac{dN_i\left(y\left(x\right);\ \sqrt{s}\right)}{dy}\ dx + \int_{x_0}^1 \frac{dN_i\left(y\left(x\right);\ \sqrt{s}\right)}{dy}\ dx, \tag{2.7}$$

where

$$x_0 = \sqrt{\frac{\langle m_\perp \rangle}{2\sqrt{s}}}. \tag{2.8}$$

Since $\frac{dN}{dy} \leq \frac{dN}{dy}\ |_{y=0}$, for the first term in the right-hand side of (2.7) (the central region term) from expressions (2.3), (2.4) we obtain

$$\int_0^{\sqrt{\langle m_\perp \rangle / \left(2\sqrt{s}\right)}} \frac{dN_p\left(y\left(x\right);\ \sqrt{s}\right)}{dy}\ dx \leq \sqrt{\frac{\langle m_\perp \rangle}{2\sqrt{s}}}\, \frac{dN_p\left(y;\ \sqrt{s}\right)}{dy}\ |_{y=0} \simeq \left(\frac{s}{s_0}\right)^{\alpha' - 0.25} \underset{s\to\infty}{\to} 0. \tag{2.9}$$

It can be shown (see **Appendix**) that in the fragmentation region defined in compliance with expression (2.5) the inclusive spectra of the DPM are presented in the form

$$\frac{dN_i\left(y(x);\ \sqrt{s}\right)}{dy} \approx \sum_c \int_x^1\ \rho_c\left(x';\ \sqrt{s}\right) \tilde{D}^{c\to i}\left(\frac{x}{x'}\right)\ dx', \quad (c \textit{ counts all possible chains}). \tag{2.10}$$

Here ρ_c is a momentum distribution function of a quark situated at an end of the chain c

$$\rho_c\left(x;\ \sqrt{s}\right) = \sum_n \frac{\sigma_n\left(\sqrt{s}\right)}{\left[\sum\limits_n \sigma_n\left(\sqrt{s}\right)\right]}\ \rho_n^c\left(x;\ \sqrt{s}\right), \tag{2.11}$$

ρ_n^c is a momentum distribution function of a quark for the process where exactly n Pomeron-cuts are involved, $\sigma_n\left(\sqrt{s}\right) / \left[\sum\limits_n \sigma_n\left(\sqrt{s}\right)\right]$ is a probability of exchange by n Pomeron-cuts, $\tilde{D}^{c\to i}$ is a function describing the fragmentation of the quark of c into the hadron i. From (2.7)–(2.10) it is easy to obtain that the following relation is valid for the partial inelasticity at high energies:

$$K_i = \sum_c \int_{x_0}^1\ dx\ \int_x^1\ \rho_c\left(x';\ \sqrt{s}\right) \tilde{D}^{c\to i}\left(\frac{x}{x'}\right)\ dx' + (\textit{terms vanishing with the rise of energy}) \tag{2.12}$$

Simple to see that the integral term in the right-hand side of (2.12) can be presented as

$$\begin{aligned}
&\int_{x_0}^1\ dx\ \int_x^1\ \rho_c\left(x';\ \sqrt{s}\right) \tilde{D}^{c\to i}\left(\frac{x}{x'}\right)\ dx' \\
&= \int_{x_0}^1\ \rho_c\left(x';\ \sqrt{s}\right)\ dx'\ \int_{x_0}^{x'}\ \tilde{D}^{c\to i}\left(\frac{x}{x'}\right)\ dx \\
&= \int_{x_0}^1\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx'\ \int_{x_0/x'}^1\ \tilde{D}^{c\to i}\left(x\right)\ dx \\
&= \int_0^1\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx'\ \int_0^1\ \tilde{D}^{c\to i}\left(x\right)\ dx \\
&\quad - \int_0^{x_0}\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx'\ \int_0^1\ \tilde{D}^{c\to i}\left(x\right)\ dx \\
&\quad - \int_{x_0}^1\ x'\rho_c\left(x';\sqrt{s}\right) dx'\ \int_0^{x_0/x'}\ \tilde{D}^{c\to i}\left(x\right) dx.
\end{aligned} \tag{2.13}$$

Let

$$M = \max_{x\in[0,1]} \tilde{D}^{c\to i}\left(x\right) \tag{2.14}$$

(very often a simple parameterization is used for the fragmentation functions: $\tilde{D}\left(x\right) = a \cdot \left(1-x\right)^b$, in this case $M = a$), then

$$\int_0^{x_0}\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx'\ \int_0^1\ \tilde{D}^{c\to i}\left(x\right)\ dx \leq M \int_0^{x_0}\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx'. \tag{2.15}$$

In the DPM the softest momentum distributions are ones of sea quarks: $\rho_{sea} \simeq 1/x$ [39,43]. So, even in the most unfavorable case one has

$$\int_0^{\sqrt{\langle m_\perp \rangle / \left(2\sqrt{s}\right)}}\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx' \times \int_0^1\ \tilde{D}^{c\to i}\left(x\right)\ dx \simeq \sqrt{\frac{\langle m_\perp \rangle}{\sqrt{s}}}. \tag{2.16}$$

Likewise,

$$\begin{aligned}
&\int_{\sqrt{\langle m_\perp \rangle / \left(2\sqrt{s}\right)}}^1\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx' \times \int_0^{\sqrt{\langle m_\perp \rangle / \left(2\sqrt{s}\right)}/x'}\ \tilde{D}^{c\to i}\left(x\right)\ dx \\
&\leq M \sqrt{\frac{\langle m_\perp \rangle}{2\sqrt{s}}} \int_{\sqrt{\langle m_\perp \rangle / \left(2\sqrt{s}\right)}}^1 \times \rho_c\left(x';\ \sqrt{s}\right)\ dx' \simeq \sqrt{\frac{\langle m_\perp \rangle}{\sqrt{s}}}
\end{aligned} \tag{2.17}$$

(in the last relation in (2.17) we took into account that the momentum distribution functions are normalized to unity: $\int_0^1\ \rho\left(x;\ \sqrt{s}\right)\ dx = 1$). From (2.12), (2.13), (2.16), (2.17) we find

$$K_i \approx \sum_c \int_0^1\ x'\rho_c\left(x';\ \sqrt{s}\right)\ dx' \times \int_0^1\ \tilde{D}^{c\to i}\left(x\right)\ dx\ . \tag{2.18}$$

Since, by definition,

$$\langle x \rangle_c = \int_0^1 x\rho_c\left(x;\ \sqrt{s}\right)\ dx, \qquad (2.19)$$

we finally obtain a simple equation for the inelasticity parameter of the DPM

$$K_i \approx \sum_c \langle x \rangle_c \int_0^1 \tilde{D}^{c\to i}(x)\ dx. \qquad (2.20)$$

It may be interesting to watch the conformity of (2.20) to the energy conservation law. Adding the fraction of energy K_0 retained by the leading particles and summing over all new produced hadrons, to obey the energy conservation one has to obtain

$$\sum_i K_i = 1. \qquad (2.21)$$

As already noted, interaction of hadrons in the DPM is divided into two stages. For the first stage, where colored chains are produced, the energy conservation is included in the relation

$$\sum_c \langle x \rangle_c = 1, \qquad (2.22)$$

and for the fragmentation stage it is taken into account by the sum rules

$$\sum_i \int_0^1 \tilde{D}^{c\to i}(x)\ dx = 1. \qquad (2.23)$$

Taking the sum in (2.20) over all possible hadrons and using two last equations, we derive

$$\sum_i K_i = \sum_i \sum_c \langle x \rangle_c \int_0^1 \tilde{D}^{c\to i}(x)\ dx$$
$$= \sum_c \langle x \rangle_c \cdot \sum_i \int_0^1 \tilde{D}^{c\to i}(x)\ dx = 1 \qquad (2.24)$$

in agreement with (2.21).

It should be mentioned that expressions similar to (2.10) (obtained here in the framework of the DPM) were widely used previously in various quark cascade models for describing the inclusive spectra in the fragmentation regions (see, for instance, review [46] and references therein). But it is also important to realize that in order to apply these expressions to the study of the inelasticity coefficient one must define what is actually meant by the fragmentation regions since the energy behavior of the resulting inelasticity may depend on this, and thus just the choice of the inclusive spectra in the fragmentation regions in the form of (2.10) does not automatically lead to such energy dependence as is prescribed by (2.20). Very often a somewhat intuitive definition of the fragmentation region as the region "say, $x > 0.1$" is used. If, for example, we assume that in the region $x < 0.1$ (or, even more generally, in the region $x < x_0$ where x_0 does not change like $1/s^{1/4}$ as the right-hand side of (2.5) but is some fixed value) the spectra continue to rise as in (2.3), (2.4), i.e. like $s^{\alpha'}$, then the inelasticity could increase with the rise of energies like $s^{\alpha'}$, too. Some other choice of the fragmentation region or/and other assumptions of the energy behavior of the spectra would bring some other energy dependence of the inelasticity (see, for example, [47]). So we see that the energy dependence of the inelasticity, predicted by (2.20), is due to the correct description of the energy behavior of the spectra in the central region and fragmentation regions as well as to the proper fragmentation region condition (2.5).

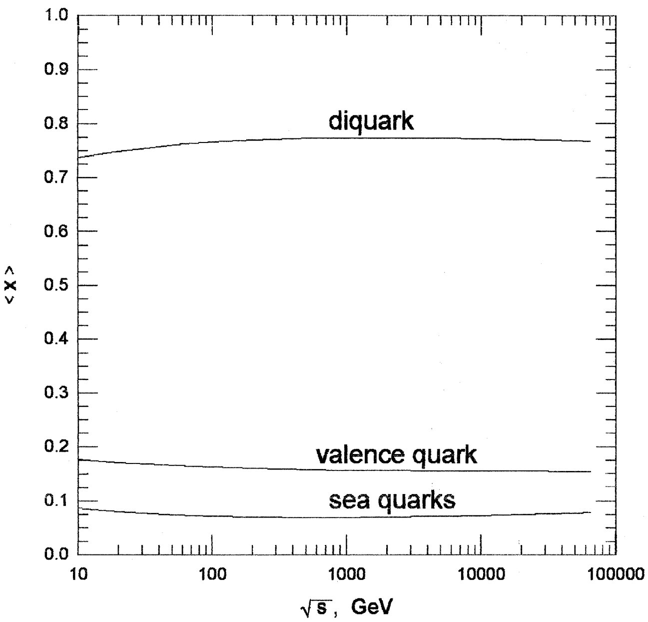

Fig. 1. $\langle x \rangle$ vs. energy for diquark, valence quark and sea quarks in nucleon

Condition (2.5) has a simple physical sense [45]. For a given c.m.s. value of rapidity y and for a chosen chain one can find the corresponding rapidity y_c in the chain's center of mass system. The region around the center $y_c = 0$ of the chain is also the central region for the chain's fragmentation. But the center of any chain can lie only on the left from $y_{\max}/2$, so for the rapidities

$$y > y_{\max}/2 \qquad (2.25)$$

there is no any chain with the central region situated near such values. It is easy to check that in terms of the variables x this relation leads to the $1/s^{1/4}$ behavior of its right-hand side, just as in (2.5). The complete condition (2.5) can be obtained from (2.25) by a more comprehensive derivation if one also takes into account the threshold effects.

Let us turn back to the (2.20) for the inelasticities. Fragmentation functions standing there can be found, for example, from positron-electron data with the help of the jet universality principle. Values of $\langle x \rangle$ can be calculated in the model itself (see (2.11), (2.19)). In Fig. 1 we show $\langle x \rangle$ for diquark, valence quark and all sea quarks for different energies. At calculations of ρ we used ρ_n from [39] with $\mu = 0.1\ GeV$, and topological cross sections σ_n with parameters defined in [41]. From Fig. 1 it is seen that $\langle x \rangle$

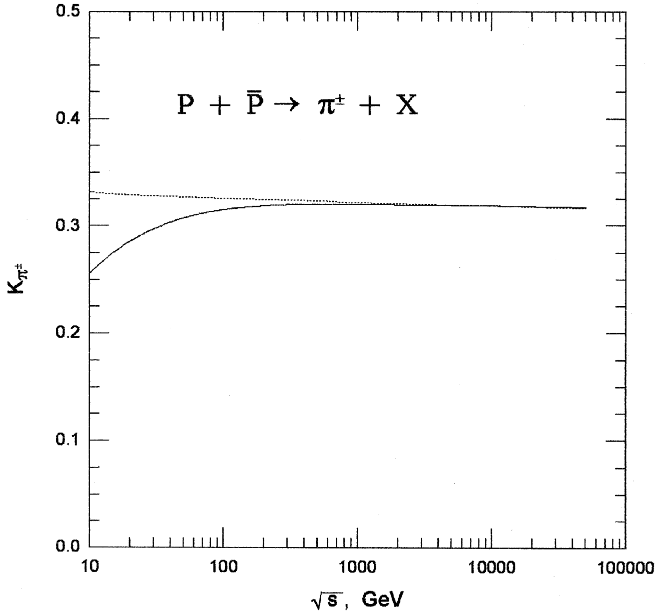

Fig. 2. Partial inelasticity for the process $p + \bar{p} \to$ charged pions + X. *Solid line* – numerical DPM's calculations, *dashed line* – calculations according to (2.20)

values change only slightly for all kinds of quarks in a wide range of energies.

In Fig. 2 the partial inelasticity numerically calculated in the DPM for the reaction $p + \bar{p} \to$ *charged pions* is presented as a function of energy (solid line). One can see from the figure that at high energies this inelasticity becomes nearly a constant. Such behavior can be readily understood from the above discussion. Since at high energies $\langle x \rangle$ values change only weakly and fragmentation functions $\tilde{D}$ used in the DPM have scaling forms, from (2.20) valid at high energies it is obvious that the partial inelasticities can not change too much with energies either. And this is just what is seen in Fig. 2.

One can also directly use (2.20) for calculations of the partial inelasticities. Relevant results are shown in Fig. 2 by the dashed line. From this figure we notice that at high energies there is a very good agreement between both curves. In fact, as it is seen from the figure, (2.20) begins to work pretty well here already from energies $50 \div 100\ GeV$.

Using an approximate relation $K_{tot} \approx \frac{3}{2} K_{\pi^\pm}$ (see, for example, [15,29,48,49]), we can estimate the total inelasticity from the data of Fig. 2. With reference to Fig. 2 we find $K_{tot} \approx 0.47$. This value is very close to one measured by *ISR*: $K_{tot} = 0.5$ [1]. The cosmic ray experiments seem to favor the total inelasticity with the value around 0.5 independent on energy, too [22,50] but because of the large errors the data of such experiments are rather inconclusive (see discussion in Sect. 1).

It should be emphasized that the results obtained in this section do not agree with the inelasticity energy behavior predictions made from some kind of a qualitative analysis in work [43]. There the elasticity coefficients $\langle E_n \rangle / E$ were presented as a decreasing function of the collision number for $P_{Lab} = 200\ GeV$. The relevant conclusion made there is as follows: since $\langle n(s) \rangle$ increases with energies, one should expect an increase of the inelasticity parameter according to the relation $K_{tot} = 1 - \langle E_{\langle n(s) \rangle} \rangle / E$. We argue, however, that because of rather wide $n-$distributions of the weights $\sigma_n(\sqrt{s}) / \left[\sum_n \sigma_n(\sqrt{s}) \right]$, it is not satisfactorily to evaluate the energy dependence of K_{tot} in such way, just trough $\langle n(s) \rangle$. Another thing that has to be also taken into account is an energy dependence of the ratio $\langle E_n \rangle / E$. Even being small, this effect should be considered properly. The calculations show that in the DPM the ratio $\langle E_n \rangle / E$ increases with the rise of energies what prevents the expected decrease of elasticity due to a smooth rise of $\langle n(s) \rangle$.

3 Scaling violation problem and the inelasticity coefficient

A very convenient parameterization of the inclusive hadronic spectra with violation of Feynman scaling was proposed by Wdowczyk and Wolfendale [19–22]

$$\frac{2E^*}{\sqrt{s}} \frac{d^2N}{dx dP_\perp} = k(s, s_0) \left(\frac{s}{s_0} \right)^\alpha f\left(x \cdot \left(\frac{s}{s_1} \right)^\alpha ;\ P_\perp \right), \tag{3.1}$$

where k and f are some functions, s_0, s_1 and α are parameters. It was found that formulae of this type (WW-formulae) can adequately describe collider data at moderate as well as at high energies, and for the reaction $p + \bar{p} \to$ *charged particles* the following relations were obtained [14,15,48]:

$$\begin{gathered} \int f\left(x \cdot \left(\tfrac{s}{s_1} \right)^\alpha ;\ P_\perp \right) dP_\perp = \left(1 - \left(\tfrac{s}{s_1} \right)^\alpha x \right)^4, \\ \alpha = 0.26, \quad s_1 = 3.4 \cdot 10^3\ \mathrm{GeV}^2; \end{gathered} \tag{3.2}$$

$$k(s, s_0) \left(\tfrac{s}{s_0} \right)^\alpha = A \cdot \left(\tfrac{s}{s_0} \right)^{\alpha'}, \tag{3.3}$$

$$A = 1.67, \quad \alpha' = 0.11 \quad s_0 = 6.3 \cdot 10^2\ GeV^2.$$

Therefore,

$$\begin{aligned} &\bar{x} \frac{dN^{p\ +\ \bar{p} \to charged\ particles}}{dx} \\ &= A \cdot \left(\frac{s}{s_0} \right)^{\alpha'} \left(1 - \left(\frac{s}{s_1} \right)^\alpha x \right)^4. \end{aligned} \tag{3.4}$$

An important thing about (3.4) is that this equation leads to the inelasticity decreasing as the energy increases:

$$\begin{aligned} K^{p\ +\ \bar{p} \to charged\ particles} &= \int \bar{x} \frac{dN^{p\ +\ \bar{p} \to charged\ particles}}{dx} dx \\ &= \frac{1}{5} A \cdot \left(\frac{s}{s_0} \right)^{\alpha'} / \left(\frac{s}{s_1} \right)^\alpha \underset{s \to \infty}{\to} 0, \end{aligned} \tag{3.5}$$

and the energy behavior $s^{\alpha'-\alpha}$ of the inelasticity coincides with the energy behavior of the parameter $k(s, s_0)$. Such behavior, however, turned out to be in a certain contradiction with the results of cosmic ray experiments. To justify the formula various assumptions were made, in some of them special h-particles were postulated which, being hadrons unobservable by collider experiments, could compensate for the decrease in the inelasticity; in others parameters α and α' were chosen close in magnitudes to slow down the fall of the inelasticity (see discussions in the works [14,15,48,49]).

In our work [45] it was found out that the DPM's predictions about the Feynman scaling violation effects for the inclusive spectra at the central region can be presented in the WW-form and the physical sense of such parameterization was discussed. At the same time, according to the results of the present work, at high energies the inelasticity of the DPM can not be a steeply decreasing function. Now we are going to show, even without any special assumptions, that these facts do not contradict each other. We had emphasized in the work [45] that according to the DPM, the WW-formula (3.4) can be applied in a rather limited interval of x variable. It was found that beginning from the value $x_0 = \sqrt{\langle m_\perp \rangle / (2\sqrt{s})}$ an alteration in the mechanisms of the production of the secondary hadrons comes and the fragmentation region behavior with rather small violation of scaling starts. So at calculating the inelasticity coefficient one should divide the full region of variable x into the central region and the fragmentation region, just as it was done in (2.7). Then the central region part of K can be expressed by the product of the height of the distributions into the width of the central region:

$$K^{centr} \simeq \left(\frac{s}{s_0}\right)^{\alpha'} \sqrt{\frac{m_\perp}{2\sqrt{s}}} \simeq \left(\frac{s}{s_0}\right)^{\alpha'-0.25}, \tag{3.6}$$

(cf. (2.9)). For the parameter α a value 0.25 was obtained in the framework of the DPM [45]. This value is very close to one of (3.2) and just coincides with the value also obtained from experiment in [2]. So

$$K^{centr} \simeq \left(\frac{s}{s_0}\right)^{\alpha'-\alpha}, \tag{3.7}$$

and this is essentially the same energy behavior as in (3.5). On the other hand, due to the approximate scaling in the fragmentation region, K^{fragm} is approximately a constant and so at high energies the main contribution to the inelasticity comes from K^{fragm}. This simple mechanism allows to retain the inelasticity almost constant and gives us a further view at the results obtained in the present work.

An example of the model with the inelasticity being an increasing function of energy is the QCD Pomeron model mentioned in Sect. 1. According to this model, the inelasticity coefficient can be presented in the form

$$K = 1 - \sum_n \frac{\sigma_n(\sqrt{s})}{\left[\sum_n \sigma_n(\sqrt{s})\right]} \langle X_n \rangle, \tag{3.8}$$

where $\langle X_n \rangle$ is an average energy fraction retained by the leading nucleon after the exchange by n Pomerons. The rise of the inelasticity in this model is due to the growth with energy (owing to the increase of σ_n) of an average number of the Pomerons involved in the reaction. Equation (3.8) can be compared with (2.20) derived in this work. Taking into account (2.11), (2.19), we obtain in the DPM

$$K_i = \sum_c \left(\sum_n \frac{\sigma_n(\sqrt{s})}{\left[\sum_n \sigma_n(\sqrt{s})\right]} \langle x_n(\sqrt{s}) \rangle_c \right) \times \int_0^1 \tilde{D}^{c\to i}(x)\ dx. \tag{3.9}$$

In contrast to (3.8), here $\langle x_n \rangle_c$ is an average fractional energy carried by a quark in hadron and so this last formula has a different sense from (3.8). The rise of σ_n with energy can result in the change of $\langle x \rangle_c = \sum_n \langle x_n(\sqrt{s}) \rangle_c \sigma_n(\sqrt{s}) / \left[\sum_n \sigma_n(\sqrt{s})\right]$ but due to the normalization (2.22) this may have no effect on K_i: if, for example, the terms $\int_0^1 \tilde{D}^{c\to i}(x)\, dx$ are alike for all c, then the corresponding inelasticity

$$\begin{aligned} K_i &= \sum_c \left(\sum_n \frac{\sigma_n(\sqrt{s})}{\left[\sum_n \sigma_n(\sqrt{s})\right]} \langle x_n(\sqrt{s}) \rangle_c \right) \times \int_0^1 \tilde{D}^{c\to i}(x)\ dx \\ &= q_i \sum_c \left(\sum_n \frac{\sigma_n}{\left[\sum_n \sigma_n(\sqrt{s})\right]} \langle x_n \rangle_c \right) = q_i \end{aligned} \tag{3.10}$$

does not depend on $\langle x \rangle_c$ at all (in (3.10) we defined $q_i = \int_0^1 \tilde{D}^{c\to i}(x)\ dx$). In the general case and even for the violated scaling this would similarly help to reduce a possible change of the inelasticity in the DPM at high energies.

Up to this moment we used the scaling fragmentation functions $\tilde{D}(x)$. It can be shown with some general assumptions of the form of $\tilde{D}$ that (2.20) remains also valid for the case of the non scaling fragmentation functions. Effects of this sort are predicted by the QCD at high energies. The energy behavior of the inelasticity in this situation depends on the non scaling corrections to the fragmentation functions. Such case is considered in the modified dual parton model [25,26].

4 Conclusions

In this work the energy dependence of the partial inelasticities was studied in the framework of the dual parton model. In order to obtain the value of the inelasticity parameter one should know the inclusive hadronic spectra in the full range of rapidities. Unfortunately, from the present experiments such data are not provided with the necessary accuracy over a wide region of energies. Such situation had naturally caused different conclusions about the possible energy behavior of the inelasticity.

The main attention of this work was paid to the study of the mechanisms which lead to a certain energy behavior of the partial inelasticities. A simple analytical formula (2.20) describing the partial inelasticities at high energies was derived. This formula has a clear physical sense. Being an average fraction of energy carried away by secondaries, the inelasticity is expressed by the product of the two factors: the average quark fractions of energy $\langle x\rangle_c$ and the full probability $\int_0^1 \tilde{D}^{c\to i}(x)\ dx$ for quark c to fragment into a hadron i.

In the DPM, as well as in some other models (see, for instance, [6]), the scheme of multiparticle production can be symbolically depicted as

$$\begin{aligned} &multiple\ production \\ &\quad = (quark\ distribution\ functions) \\ &\qquad \otimes (universal\ fragmentation)\,. \end{aligned}$$

Here we would like to emphasize once more that it does not undeniably imply the possibility of the presentation of the partial inelasticities in the form (2.20). In the DPM the presentation of the inelasticities in the factorized form of (2.20) is not true for the central region where fragmentation from both projectile hadron and target hadron takes place. For the central region one has the growth of the inclusive spectra as $s^{\alpha'}$ where $\alpha' \approx 0.11$, and in order to find the inelasticity as a function of energy one must clearly determine the width of the central region. It proved to be [44] that in the DPM in the area (2.5) the fragmentation region behavior with relatively small violation of scaling is seen and in this area the relation (2.10) is valid for the inclusive spectra (see **Appendix**). The central region part of the DPM's inelasticity is then a decreasing function of energy and its energy behavior agrees very well with one obtained from the WW-formula (3.4). We argue, however, that at high energies the main contribution to the inelasticity comes from the fragmentation region and the partial inelasticities can be found using (2.20). As a matter of fact, for the reaction $p\ +\ \bar{p} \to charged\ pions$ this approximate formula begins to work already from energies $\sqrt{s} = (50 \div 100)\ GeV$. Estimations show that K_{tot} obtained in the DPM is about 0.47. This value is very close to the results of measurements at *ISR*.

Here we investigated the inelasticity parameter defined according to (2.1). Similar definition also have the so-called Z-factors

$$Z_i = \int_0^1 x^\gamma \frac{dN_i}{dx}\ dx, \tag{4.1}$$

where power γ comes from the primary energy spectrum. It can be shown that in the DPM at high energies for the Z-factors the expression analogous to (2.20) takes place as well

$$Z_i \approx \sum_c \langle x^\gamma\rangle_c \int_0^1 \tilde{D}^{c\to i}(x)\ dx. \tag{4.2}$$

A more detailed study of the energy behavior of the Z-factors in the framework of the dual parton model will be a subject of our separate work.

A Appendix

Here we will show that in the dual parton model a presentation of the inclusive spectra in the form (2.10) is valid all over the region (2.5). According to the DPM, the inclusive spectra of hadron i can be found from the following expressions ([37–40, 43]):

$$\begin{aligned} \frac{dN_i\left(y(x);\ \sqrt{s}\right)}{dy} &= \sum_n \frac{\sigma_n\left(\sqrt{s}\right)}{\left[\sum_n \sigma_n\left(\sqrt{s}\right)\right]} \sum_{c_{q_1;q_2}} \int_x^1 dx_1\ \rho_n^{q_1}(x_1) \\ &\times \int_r^1 dx_2\ \ \rho_n^{q_2}(x_2)\, \tilde{D}^{q\to i}(z)\,, \\ z &= \frac{2\langle m_\perp\rangle}{\sqrt{x_1 x_2 s}} \sinh\left(y - \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right)\right). \end{aligned} \tag{A.1}$$

Here q_1 and q_2 are the quarks situated at the different ends of the chain c (the sum with the index $c_{q_1;q_2}$ in (A.1) goes over all possible chains), and $r = 2\langle m_\perp\rangle/\sqrt{s}$ originates from a natural threshold on the production of the secondary hadron with the transverse mass $\langle m_\perp\rangle$. The fragmentation function $\tilde{D}^{q\to i}(z)$ $(= \bar{z}D^{q\to i}(z)$, q is either q_1 or $q_2)$ is taken in the DPM according to the relation:

$$\tilde{D}^{q\to i}(z) = \begin{cases} \tilde{D}^{q_1\to i}(z) & \text{if}\ \ z > 0 \\ \tilde{D}^{q_2\to i}(|z|) & \text{if}\ \ z < 0. \end{cases} \tag{A.2}$$

For the following it is convenient to introduce a new variable g

$$g = e^{2y}. \tag{A.3}$$

It is easy to see that in the considered region of x a simple relation between x and g takes place at high energies

$$g \approx \left(\frac{x\sqrt{s}}{\langle m_\perp\rangle}\right)^2. \tag{A.4}$$

For the parameter z from (A.1) we obtain

$$z = \frac{\langle m_\perp\rangle}{x_1 x_2 \sqrt{s}}\left(x_2\sqrt{g} - \frac{x_1}{\sqrt{g}}\right). \tag{A.5}$$

It is apparent that at

$$gr > 1 \tag{A.6}$$

$x_2\sqrt{g}$ is always greater than $x_1/\sqrt{g}$ for any x_1 and x_2 from the ranges of integration in (A.1). Expression (A.6) can be written, in view of (A.4), as

$$x > \sqrt{\frac{\langle m_\perp\rangle}{2\sqrt{s}}}, \tag{A.7}$$

what coincides with the definition of the fragmentation region, given in the main text (see (2.5)). In this region one always has $z > 0$ and so, in compliance with (A.2), here $\tilde{D}^{q\to i}(z) = \tilde{D}^{q_1\to i}(z)$. The expression (A.1) for the inclusive spectra assumes now the form

$$\frac{dN_i\left(y\left(x\right);\ \sqrt{s}\right)}{dy} = \sum_n \frac{\sigma_n\left(\sqrt{s}\right)}{\left[\sum\limits_n \sigma_n\left(\sqrt{s}\right)\right]} \times \sum_{c_{q_1;q_2}} \int_x^1 dx_1\ \rho_n^{q_1}\left(x_1\right) \int_r^1 dx_2\ \rho_n^{q_2}\left(x_2\right) \times \tilde{D}^{q_1\to i}\left(\frac{x}{x_1} - \frac{1}{g}\frac{x}{x_2}\right). \quad \text{(A.8)}$$

Let us expand the function $\tilde{D}^{q_1\to i}$ from the right-hand side of (A.8) as a power series in $\frac{1}{g}\frac{x}{x_2}$ (to simplify notations we will omit for the present the symbol $q_1 \to i$ at the function $\tilde{D}$)

$$\tilde{D}\left(\frac{x}{x_1} - \frac{1}{g}\frac{x}{x_2}\right) = \sum_{k=0}^{\infty} \frac{\left(-1\right)^k}{k!}\frac{1}{g^k}\left(\frac{x}{x_2}\right)^k \tilde{D}'^{(k)}\left(\frac{x}{x_1}\right). \quad \text{(A.9)}$$

Since $x_2 \geq r$, each successive term in this expansion has at least one extra power of $x/\left(gr\right)$. From (A.4) it follows that in the region (2.5) this does not exceed $\sqrt{m_\perp/\left(2\sqrt{s}\right)}$. So at high energies one can restrict the sum (A.9) to the first leading summand. In this approximation,

$$\frac{dN_i\left(y\left(x\right);\ \sqrt{s}\right)}{dy} \approx \sum_n \frac{\sigma_n\left(\sqrt{s}\right)}{\left[\sum\limits_n \sigma_n\left(\sqrt{s}\right)\right]} \times \sum_{c_{q_1;q_2}} \int_x^1 \rho_n^{q_1}\left(x_1\right)\tilde{D}^{q_1\to i}\left(\frac{x}{x_1}\right)\ dx_1 \quad \text{(A.10)}$$

what differs from (2.10) in notations only (see (2.11)). From the derivation it is clear that $x_0 = \sqrt{m_\perp/\left(2\sqrt{s}\right)}$ is the minimal value of Feynman's variable x for which the presentation of the inclusive spectra in the form (A.10) still takes place. For any lesser value of x the contribution from the fragmentation of the quark q_2 has to be taken into account.

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